Bunching Techniques to Identify Threshold Effects

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May 12th, 2025

Introduction

- The term "Bunching" refers to behavioural responses of individuals/firms
- It first appeared in the tax literature to study responses to taxes (Saez, 2010)

  $\rightarrow$ now bunching techniques are used in other settings as well
- Two-fold objective of bunching designs:
  1. providing non-parametric evidence of a behavioural response (to taxes)
  2. mapping responses to structural parameters, useful to predict effects of (tax) policy changes (not easy)
- Bunching methods related to Regression Discontinuity (RD) and Kink (RK) designs

  $\rightarrow$ the difference lies in whether the relevant variable can be manipulated or not

Responses to What: Types of discontinuities in the tax literature

- Bunching as a response to two types of discontinuity in the choice set:
  1. kinks: discrete changes in the slope of choice sets (Chetty et al., 2011; Saez, 2010)

     $\rightarrow$ discontinuity in the marginal tax rate (MTR)
  2. notches: discrete changes in the level of choice sets (Kleven & Waseem, 2013)

     $\rightarrow$ discontinuity in the average tax rate (ATR)
- The main focus for today is notches: more common in non-tax related settings

Example of notches: Kleven and Waseem (QJE 2013)

FIGURE V

Personal Income Tax Schedules in Pakistan

Bunching Design Applications: some examples

1. Labor supply/Reporting responses to taxation
   Bastani and Selin (2014), Harju et al. (2019), Kleven and Waseem (2013), Le Maire and Schjerning (2013), and Saez (2010)
   - e.g. estimating the elasticity of taxable income (ETI), income shifting, costs of complying with specific tax regimes, optimisation frictions
2. Social Insurance and Welfare Programs
   Chetty et al. (2013), Khoury (2023), Mortenson and Whitten (2020), and Seibold (2021)
   - e.g. responses to the EITC, lay-offs responses to UI benefits discontinuity at job tenure threshold, retirement behaviour responses to framing of the pension system
3. Housing market transactions
   (Best & Kleven, 2018; Kopczuk & Munroe, 2015; Slemrod et al., 2017)
   - e.g. studying responses of transactions affected by new taxes on value property above certain thresholds, effects temporary tax breaks on housing market activity

Roadmap

Theory: Kinks and Notches in the Tax Literature

Bunching Estimation: Standard Method for the Counterfactual Distribution

Identification Issues

Alternative Method: Control Group Bunching Design

Two Applications in the Literature
- "Compliance Costs vs. Tax Incentives: Why Do Entrepreneurs Respond to Size-based Regulations?" by Harju, J., Matikka, T. and Rauhanen, T. (JPubE '19)
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(Tax) Kinks

- For individual with ability n, define preferences over before and after tax income,

$$u(z - T(z), z/n) \tag{1}$$

- Assumption: smooth ability distribution $f(n)$, preferences u, and tax system $T(z)$

  $\rightarrow$ smooth earnings distribution $h_0(z)$

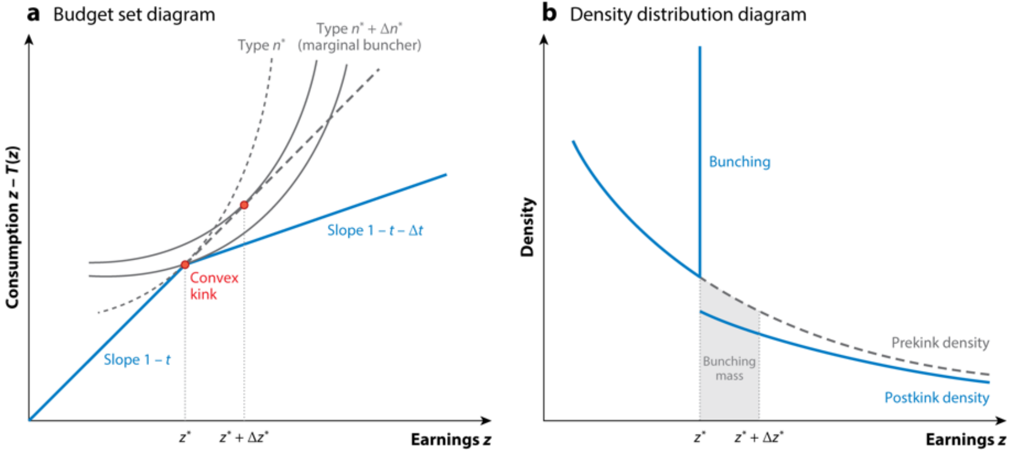
- consider a linear tax system $T(z) = t \cdot z$

  $\rightarrow$ and a kinked tax function: $T(z) = t \cdot z + \Delta t \cdot (z - z^*) \mathbb{1}(z > z^*)$

- agents with earnings between z and $z^* + \Delta z^*$ bunch at the kink

  $\rightarrow$ workers with pre-kink earning above $z^* + \Delta z^*$ also reduce earnings

What is a tax kink



Kleven HJ. 2016.
Annu. Rev. Econ. 8:435–64

Homogenous Elasticity with a small kink

- The bunching approach connects the response $\Delta z^*/z^*$ of the marginal buncher to the (compensated) earnings elasticity

$$e = \frac{\Delta z^*/z^*}{\Delta t/(1-t)} \tag{2}$$

- and to the extent of the bunching mass:

$$B = \int_{z^*}^{z^*+\Delta z^*} h_0(z)dz \simeq h_0\left(z^*\right)\Delta z^* \tag{3}$$

- the approximation requires the counterfactual distribution $h_0(z)$ to be constant in the bunching segment $(z^*, z^* + \Delta z^*)$

Homogenous Elasticity with a large kink

- Need to specify preferences, e.g. $u = z - T(z) - \frac{n}{1+1/e} \cdot \left(\frac{z}{n}\right)^{1+1/e}$ (No inc. effects)
- Individuals choose before tax earning: $z = n(1-t)^e$
- The marginal buncher is optimising both right above the kink and on the pre-kink budget line
  - pre-kink budget line: $z^* = (n + \Delta n)(1-t)^e$
  - right above the kink: $z^* + \Delta z^* = (n + \Delta n)(1 - t - \Delta t)^e$
- Dividing the two conditions above and further manipulation gives:

$$e = -\frac{\log\left(1 + \Delta z^*/z^*\right)}{\log(1 - \Delta t/(1-t))} \tag{4}$$

which generalises the previous elasticity formula (2)

(Tax) Notch (Kleven & Waseem 2013)

- consider a discrete increase Δt in the average tax rate t from earning z^*
  $T(z) = t \cdot z + \Delta t \cdot z \cdot \mathbb{1}(z > z^*)$
- agents with pre-notch earnings between z and $z^* + \Delta z^*$ bunch at the notch

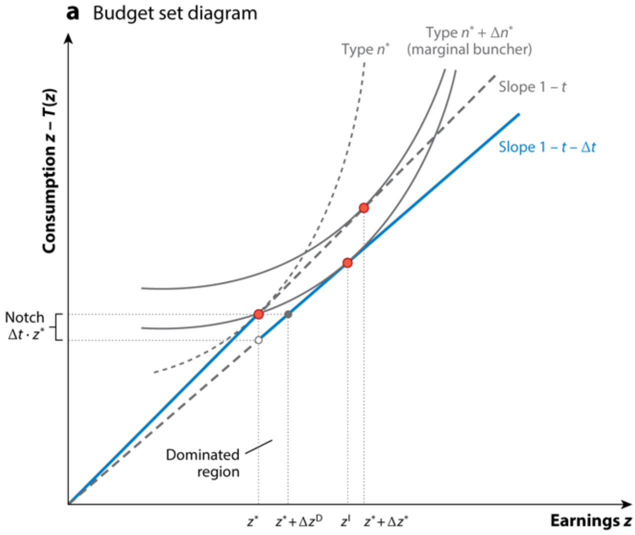
  $\rightarrow$ workers with pre-kink earning above $z^* + \Delta z^*$ also reduce earnings
- the individual with pre-notch earnings equal to $z^* + \Delta z^*$ is the marginal buncher

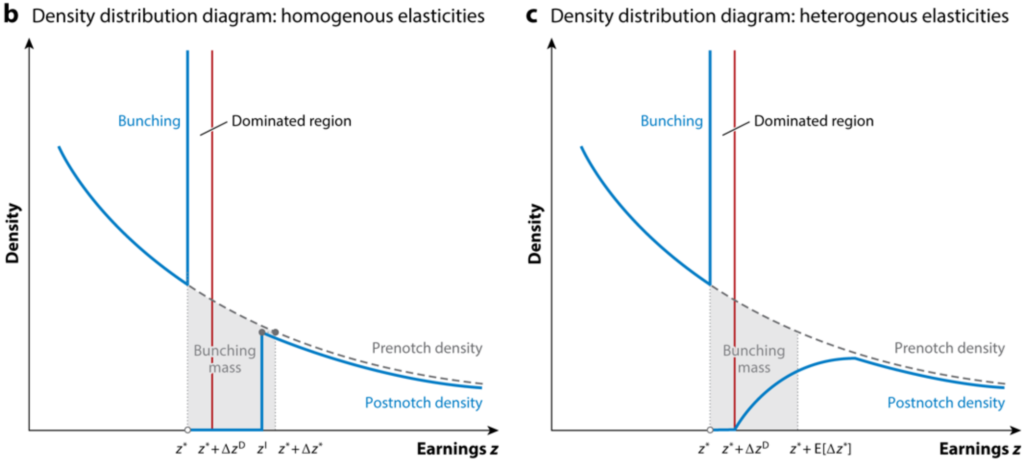
  $\rightarrow$ indifferent between notch point and best interior point after the tax change z^I
- moreover, the notch creates a dominated region of earnings $(z^*, z^* + z^D)$

  $\rightarrow$ bunching at the threshold increases both consumption and leisure

  $\rightarrow$ no incentive to stay there, in the case of no optimitisation frictions

Kleven (Annu. Rev. Econ. 2016)





Kleven HJ. 2016.
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(Tax) Notch - elasticity

- We exploit an indifference condition of the marginal buncher individual

$$\frac{1}{1+\Delta z^*/z^*} - \frac{1}{1+1/e}\left[\frac{1}{1+\Delta z^*/z^*}\right]^{1+1/e} - \frac{1}{1+e}\left[1-\frac{\Delta t}{1-t}\right]^{1+e} = 0 \quad (5)$$

- this expression characterises the relationship between
  - percentage earnings response $\Delta z^*/z^*$, estimable with bunching using (3)
  - percentage change in the average net-of-tax rate $\frac{\Delta t}{1-t}$ (institutional parameter)
  - compensated elasticity e

Derivation Indifference condition (Kleven & Waseem 2013)

- the marginal buncher is indifferent between bunching at the notch with utility

$$u^* = (1-t)z^* - \frac{n^* + \Delta n^*}{1 + 1/e}\left(\frac{z^*}{n^* + \Delta n^*}\right)^{1+1/e}$$

  and remaining at the best interior point above the notch z^I with utility u^I

- exploiting the FOC $n^* + \Delta n^* = \frac{z^* + \Delta z^*}{(1-t)^e}$, we can rewrite utility at z^I as

$$u^I = \left(\frac{1}{1+e}\right)(n^* + \Delta n^*)\,(1 - t - \Delta t)^{1+e}$$

- and then we can impose $u^* = u^I$ and obtain condition (5)

Heterogeneous Elasticities and Optimisation Frictions

- Some individuals may fail to bunch due to adjustment costs and/or inattention, s.t.

$$B = B(e, x, \phi)$$

$\rightarrow$ observed elasticity $<$ structural elasticity

- Observed B can result from different (e, ϕ) combinations
- Kleven and Waseem (2013) estimate the frictionless response $E\left[\Delta z_e^*\right]$ from

$$B = \int_e \int_{z^*}^{z^*+\Delta z_e^*} (1-\beta(z,e,\phi))\hat{h}_0(z,e)dzde \simeq h_0\left(z^*\right)\left(1-\beta^*(\phi)\right)E\left[\Delta z_e^*\right] \quad (6)$$

by exploiting the share of non optimisers β in the dominated region $\left(z^*, z^* + \Delta z^D\right)$

Reference Points

- Public policies often rely on specific (eligibility) thresholds
- these thresholds can become focal (reference) points s.t.

$$B = B(e, x, \phi, r)$$

- Hence, reference points amplify bunching on top of financial incentives

  $\rightarrow$ observed elasticity $>$ structural elasticity

  $\rightarrow$ need to find at least three data moments to pin down (e, ϕ, r)
- some examples of round-number bunching:
  - round number reporting in income (Kleven & Waseem, 2013), house prices from transaction data (Best & Kleven, 2018)
  - policy induced focal points (Seibold, 2021)

Policy induced focal points: Seibold (2021)

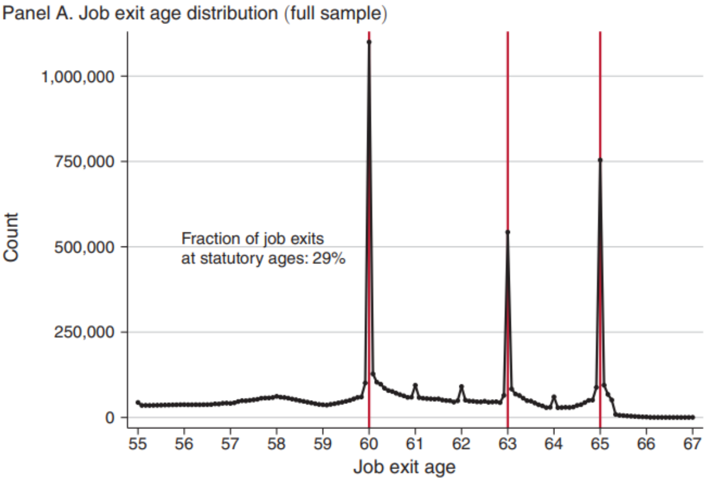


Figure: Pooled distribution of retirement ages for all workers born between 1933 and 1949

Roadmap

Introduction

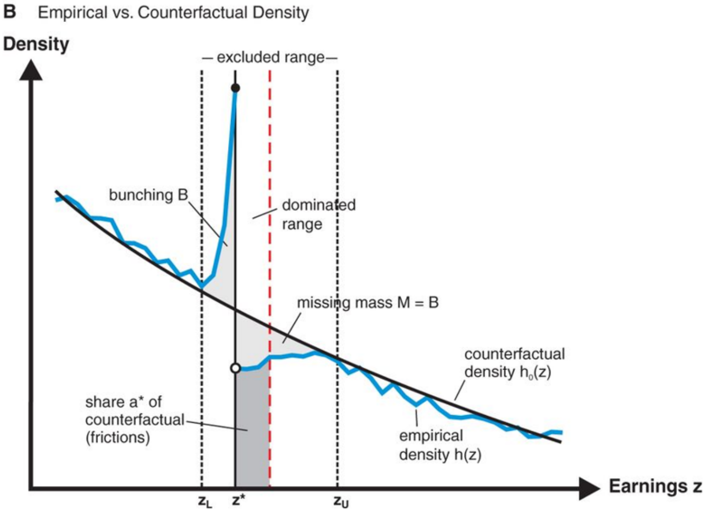
- To estimate the bunching mass we need to compare the empirical distribution with an appropriate counterfactual

  $\rightarrow$ i.e. what the distribution would have looked like without the discontinuity

- We will consider two methodologies:

  1. standard method: developed by Chetty et al. (2011) and Saez (2010) for kinks, and by Kleven and Waseem (2013) for notches
  2. control group bunching design

Bunching Estimation: standard method (Kleven & Waseem, 2013)



Bunching Estimation Methodology (Kleven & Waseem '13)

- Fit a flexible polynomial to the observed distribution, excluding data in a range around the cut-off z^*

$$c_j = \sum_{i=0}^{p} \beta_i \cdot (z_j)^i + \sum_{i=z_L}^{z_U} \gamma_i \cdot 1\left[z_j = i\right] + \nu_j \tag{7}$$

- then, extrapolate the fitted distribution to the cut-off using

$$\hat{c}_j = \sum_{i=0}^{p} \hat{\beta}_i \cdot (z_j)^i \quad \text{for } j \in [z_L, z_U]$$

- choose z_U s.t. Excess bunching (B) equals missing mass (M)

$$\hat{B} = \sum_{j=z_L}^{z^*} (c_j - \hat{c}_j) \qquad \hat{M} = \sum_{j>z^*}^{z_U} (\hat{c}_j - c_j)$$

To keep in mind

- lower bound of the excluded region z_B: choose bin where bunching behaviour starts

  $\rightarrow$ upper bound excluded region z_U is obtained by setting $B = M$
- choice of the order of the polynomial

  $\rightarrow$ try multiple options to check that results are robust
- if omitting n bins below z^*, bunching coefficient tells us how strong bunching is:

$$b = \frac{B}{n^{-1} \sum_{j>z_L}^{z^*} \hat{c}_j}$$

Kleven and Waseem 2013

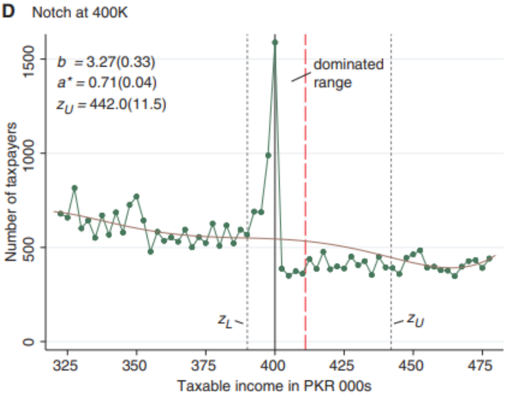


FIGURE VI

Bunching Estimation with round number bunching

- Add round number fixed effects

$$c_j = \sum_{i=0}^{p} \beta_i \cdot (z_j)^i + \sum_{r \in R} \rho_r \mathbb{1}\left[\frac{z_j}{r} \in \mathbb{N}\right] + \sum_{i=z_L}^{z_U} \gamma_i \cdot \mathbb{1}\left[z_j = i\right] + \nu_j$$

  where $R = \{1K, 5K \ldots\}$ is a vector of round-number multiples rounding
- Excess bunching defined as: $\hat{B} = \sum_{j=z_L}^{z^*} (c_j - \hat{c}_j)$ with

$$\hat{c}_j = \sum_{i=0}^{p} \hat{\beta}_i \cdot (z_j)^i + \sum_{r \in R} \rho_r \mathbb{1}\left[\frac{z_j}{r} \in \mathbb{N}\right] \text{ for } j \in [z_L, z_U]$$

- Alternative strategy: drop observations with earnings being multiples of 500 or 1K euros, then estimate (7)

Bunching Estimation with Extensive Margin Responses

- A big tax change Δt above z^* might generate extensive margin responses

  $\rightarrow$ the estimated missing mass might be bigger than excess mass
- estimating the counterfactual using bins above z_U does not represent the full counterfactual stripped of all behavioural responses to the notch
- if extensive margin responses affect the upper bracket, estimate (7) just below the threshold (Kleven, 2016)

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- Theory: Kinks and Notches in the Tax Literature
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- Identification Issues
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- Two Applications in the Literature
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Issue 1: counterfactual density

- The smoothness assumption for the counterfactual $h_0(z)$ is not enough to pin down a unique behavioural response Δz^* (Blomquist et al., 2021)

$$B = \int_{z^*}^{z^*+\Delta z^*} h_0(z)dz$$

$\rightarrow$ control-group bunching method might help here

- Also notice that the assumption of smoothness might be violated if
  - other policy changes at the same threshold
  - threshold is a reference point

$\rightarrow$ take care of round-number bunching in the estimation

$\rightarrow$ obtain additional bunching estimation by exploiting the variations in the size of the discontinuity

Issue 2: mapping between Δz^* and elasticity e

- In the simplest model (Static, frictionless, deterministic, perfect compliance and no behavioral biases):

$$B = \int_{z^*}^{z^*+\Delta z^*} h_0(z)dz, \qquad \Delta z^* = f(e, x).$$

- However, other factors Φ can influence responses: $\Delta z^* = f(e, \Phi, x)$ including
  - Evasion and avoidance
  - Income uncertainty
  - Lumpiness (indivisibility of hours)
  - Adjustment costs, inattention and misperception
  - Reference dependence

- Without assumptions or evidence on Φ, we cannot pin down the elasticity e (Kleven, 2016)

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Control group bunching design

- Following the example of Devereux et al. (2014)
- Key idea: to exploit a control group of taxpayers to build the counterfactual

  → e.g. consider individuals of a category not affected by the policy change at the threshold

  → or consider individuals around the threshold in the pre-reform period
- rescale the distributions of the chosen control group to match the number of individuals around the threshold in the period of interest

Control group bunching design: estimation

- Consider a finite interval $[z_{min}, z_{max}]$
- let $c_{j,t_{before}}$ be the number of individuals grouped in bin j, before the policy change
- compute the relative frequency $p_{j,t_{before}}$ of individuals in each bin for each pre-reform year

$$p_{j,t_{before}} = \frac{c_{j,t_{before}}}{\sum_{i=z_{min}}^{z_{max}} c_{i,t_{before}}}.$$

- Define the counterfactual frequency in each bin as the weighted average of the relative frequency across n pre-reform years (population weights $w_{t_{before}}$)

  $\rightarrow$ normalised by the number of individuals in the post-reform empirical distribution

$$\hat{c}_j = \sum_{t_{before}} w_{t_{before}} p_{j,t_{before}} \cdot \sum_{i=z_{min}}^{z_{max}} c_{i,t_{after}}.$$

Bunching in practice

- R: package "bunching" developed by Mavrokonstantis (2019); "bunchr" by Itai Trilnick https://cran.r-project.org/web/packages/bunchr/bunchr.pdf
- STATA: package "bunching developed by Bertanha et al. (2022), "rfbunch", "polbunch" (in progress) by Martin E. Andresen https://sites.google.com/site/martineckhoffandresen/software

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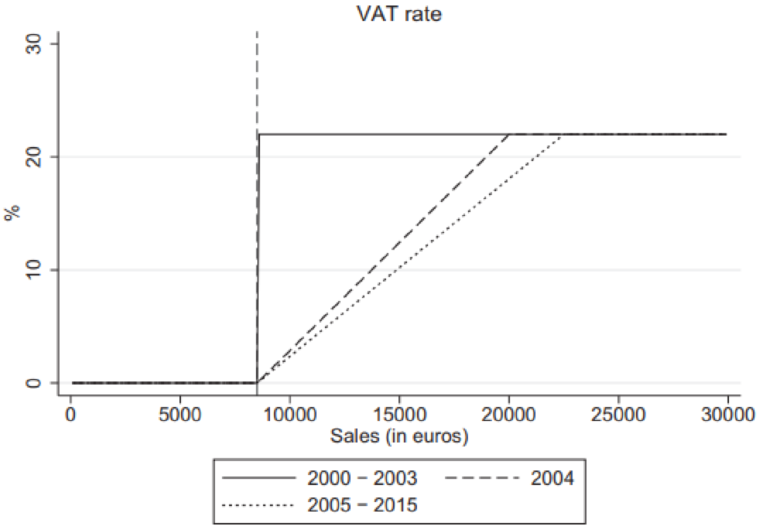
Harju, Matikka and Rauhanen, JPubEcon 2019

- They study (revenue) responses to the VAT registration threshold using administrative data on all Finnish firms
- Below the revenue threshold (€8,500), entrepreneurs do not file for VAT
  - lower compliance costs if not filing VAT reports
  - possibly lower tax liability if incidence of VAT is partly on the entrepreneurs
- Analysis: standard bunching estimation + new indifference conditions for the marginal buncher under each regime
- Three different regimes:
  1. Before 2004: tax notch at €8,500
  2. 2004-2009: tax kink with VAT-relief scheme (lower tax incentive to bunch at the threshold)
  3. 2010-2015: reform lowering compliance costs for entrepreneurs filing VAT reports

Harju, Matikka and Rauhanen, JPubEcon 2019

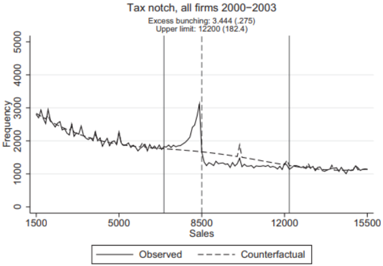


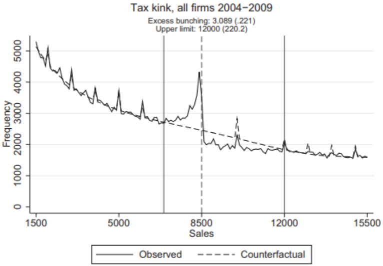
Harju, Matikka and Rauhanen, JPubEcon 2019

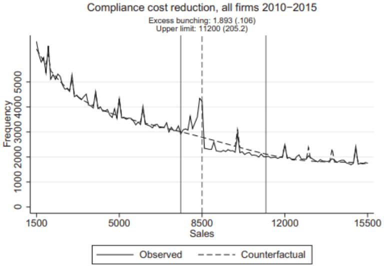
- Changes to the tax and filing policy over time are exploited for indentification
- Three data moments to exploit to estimate three parameters:
  - revenue elasticity e
  - compliance costs due to the VAT tax filing (in 2004-2009 and 2010-2015)
- Findings: responses are mainly driven by lower compliance costs, not tax incentives

  $\rightarrow$ including compliance costs (€1,300) decreases elasticity from 0.55 to 0.016

Harju, Matikka and Rauhanen, JPubEcon 2019







Seibold (AER 2021)

- This paper studies the large concentration of retirement behaviour around statutory retirement ages

  → estimating bunching responses to 644 pension benefit discontinuities
- Discontinuities exploited: contribution notches, kinks at statutory retirement ages, disability pension
- Three statutory retirement ages: ERA, FRA, NRA
- On average, responses to statutory retirement ages are seven times larger than to pure financial incentives
- Framing of statutory retirement ages can explain the observed responses

  → suggesting that changing retirement ages can influence retirement behaviour

Seibold (AER 2021)

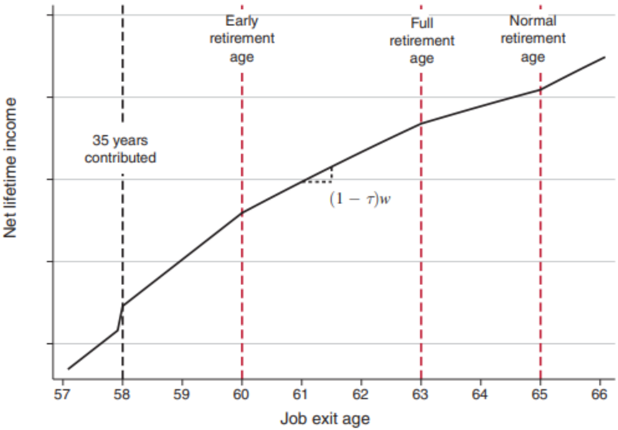
TABLE 1—PATHWAYS INTO RETIREMENT

Pathway	Required contributions	Other requirements	Statutory retirement ages (cohort 1941) Early	Full	Normal	Share of Sample (percent)
Regular	5 years	—	65	65	65	5
Long-term insured	35 years	—	63	65	65	19
Women	15 years 10 years full	Female	60	61	65	32
Unemployed/part-time	15 years 8 years full	Unemployed or in part-time work before retirement	60	64	65	20
Invalidity	35 years	Disability status	60	60	65	12
Disability	5 years 3 years full	Stricter disability status	—			11

Notes: The table presents an overview of pathways into retirement. For each pathway, statutory retirement ages are shown for a worker born in January 1941. Note that statutory ages vary over the sample period as shown in online Appendix Figure A2. The disability pathway does not have any statutory ages. For the unemployed/part-time pathway, unemployment for at least 1 year or old-age part-time work for at least 2 years after age 58 is required. For the invalidity pathway, an officially recognized disability of a certain degree is required; the disability pathway entails a stricter disability requirement, such that the worker is not able to work more than 3 hours a day in any job. *full* contribution years exclude periods where contributions are paid voluntarily. The last column shows the share of workers in each pathway in the full individual sample.

Seibold (AER 2021)

Panel B. Stylized lifetime budget constraint



Seibold (AER 2021)

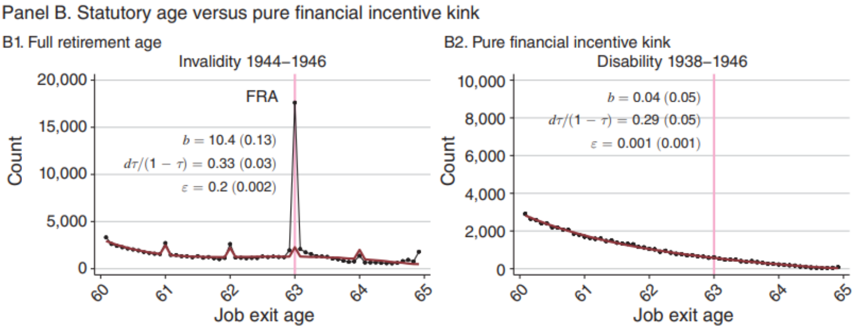


Figure 3. Bunching at Specific Discontinuities

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